Let $T_n(x)$ be the n^{th} degree Taylor polynomial for $f(x) = e^{2x}$ about $x = 0$. **SCORE: _____ / 8 PTS**

[a] If $-\frac{1}{3} \le x \le \frac{1}{3}$, what is the maximum difference between $f(x)$ and $T_4(x)$ according to Taylor's Inequality/Remainder Theorem ? Write your answer as a fraction, **NOT** as a decimal. Your answer may use e.

$$|f^{(5)}(x)| = \underbrace{|32e^{2x}|}_{1} \le 32e^{\frac{2}{3}}$$

$$|f(x) - T_4(x)| \le \frac{32e^{\frac{2}{3}}}{5!}|x|^5 \le \overbrace{\frac{32e^{\frac{2}{3}}}{\underbrace{3^5}_{1\frac{1}{2}} \cdot \underbrace{5!}_{1}}}^{1\frac{1}{2}}$$

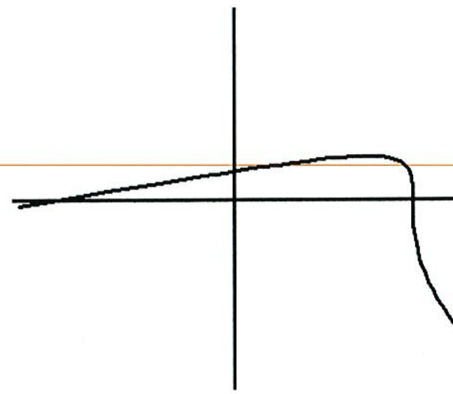
[b] What is the maximum difference between $f(-\frac{1}{3})$ and $T_4(-\frac{1}{3})$ according to the Alternating Series Estimation Theorem ? Write your answer as a fraction, **NOT** as a decimal.

$$e^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^n}{n!} \longrightarrow e^{-\frac{2}{3}} = \sum_{n=0}^{\infty} \frac{(-\frac{2}{3})^n}{n!}$$

$$|f(-\tfrac{1}{3}) - T_4(-\tfrac{1}{3})| \le \frac{(-\frac{2}{3})^5 \;]\, 2}{5! \;]\, 1}$$

Find the area between the x – axis and the curve $\begin{array}{l} x = 4 - t^3 \\ y = 2t - t^2 \end{array}$ in the graph shown on the right. **SCORE: _____ / 8 PTS**

Do NOT find a Cartesian/rectangular equation for the curve.



X-INT OCCUR WHEN $y = 0$

1 $\underline{2t - t^2 = 0} \longrightarrow t = 0, 2$

$(x, y) = (4, 0)$ @ $t = 0$

$(-4, 0)$ @ $t = 2$

$$\underbrace{\boxed{\int_2^0}}_{2} \underbrace{(2t - t^2)(-3t^2)\,dt}_{2} = \int_2^0 (-6t^3 + 3t^4)\,dt$$

$$= 2\; \underline{\left(-\tfrac{3}{2}t^4 + \tfrac{3}{5}t^5\right)}\Big|_2^0$$

$$= \tfrac{3}{2} \cdot 2^4 - \tfrac{3}{5} \cdot 2^5 = 24 - \tfrac{96}{5} = \underbrace{\tfrac{24}{5}}_{1}$$

MUST BE IN CORRECT ORDER – ONLY 1 POINT IF LIMITS REVERSED

Find a Cartesian/rectangular equation for the curve $\begin{matrix} x = \frac{1}{3}\ln 2t \\ y = 5t^4 \end{matrix}$.

SCORE: _____ / 4 PTS

$3x = \ln 2t$

$e^{3x} = 2t$

2 $\underline{t = \frac{1}{2}e^{3x}}$

$y = 5(\frac{1}{2}e^{3x})^4$

2 $\underline{y = \frac{5}{16}e^{12x}}$

Find the equation of the tangent line to the curve $\begin{matrix} x = t^2 + t \\ y = t^3 - 5t + 1 \end{matrix}$ at the point $(2, 3)$.

SCORE: _____ / 6 PTS

<u>For convenience, write the equation in point-slope form, NOT slope-intercept form.</u>

$\boxed{\frac{dy}{dx} = \frac{3t^2 - 5}{2t + 1}}$ 2

1 $\underline{t^2 + t = 2}$

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0$

$t = -2, \cancel{1}$

$(x,y) = (2,3) @ t = -2$

$\cancel{= (2,-3) @ t = 1}$

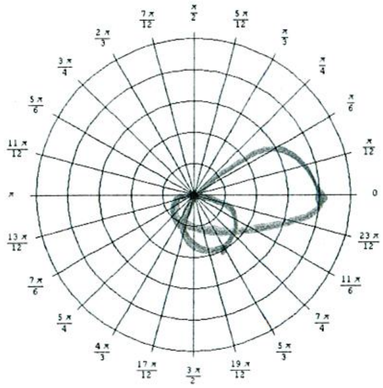
$\left.\frac{dy}{dx}\right|_{t=-2}$ 1 $= \frac{7}{-3}$ 1

$\underline{y - 3 = -\frac{7}{3}(x - 2)}$

1

The graph on the right shows r as a function of θ in Cartesian coordinates.
Use it to sketch the corresponding polar graph $r = f(\theta)$ on the polar graph paper below.

SCORE: _____ / 4 PTS



GRADED BY ME